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EFFECTIVENESS OF ACIDIZING OF THE WELLBORE ZONE TAKING INTO ACCOUNT THE CAPILLARY LOCKING OF FORMATION WATER

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The results of calculations of immiscible fluid flow to an operating well are compared for various formation parameters and states of the wellbore zone. Steady-state and unsteady filtration regimes are considered. The calculations show that acidizing of the wellbore zone increases the well flow rate.

Key words: *two-phase filtration, acidizing, numerical modeling.*

When a HCl/HF mixture is injected into a formation, it reacts with the SiO_2 sandy fraction. As a result, the pore radii increase, leading to an increase in the porosity m and permeability (permeability coefficient) of the formation k (k_f).

As shown in [1], the linearized model problem of an increase in the radius of an individual capillary with an aggressive fluid flowing inside it and reacting with the inner surface reduces to solving the differential equations

$$\frac{\partial c_1}{\partial x_1} = -c_1 + r_1 + 1, \qquad \frac{\partial r_1}{\partial x_2} = a_1(1 - c_1) \tag{1}$$

subject to the boundary conditions

$$x_1 = 0,\ x_2 \geqslant 0: \quad c_1 = 0; \qquad x_2 = 0,\ x_1 \geqslant 0: \quad r_1 = 0. \tag{2}$$

Here $x_1 = ax/R_0$ and $x_2 = 2At/r_0 - ax/R_0$ are the dimensionless characteristic variables, x is the physical coordinate, t is time, A is the rate constant of the second-order chemical reaction, $a_1 = C_0/(2\rho_1)$, C_0 is the initial concentration of the reactive fluid, R_0 is the initial radius of the capillary, ρ_1 is the density of the reactive component of the capillary surface, r_1 and c_1 are the small relative variations in the radius and fluid concentration, respectively, $a_1 = 2A/(\pi v_0)$, and v_0 is the fluid flow velocity.

The Goursat problem (1) and (2) is easy to solve using a Laplace transform. The solution has the form

$$c_1(x_1, x_2) = 1 - \mathrm{e}^{-x_1}\, J_0(2\sqrt{a_1 x_1 x_2}), \qquad r_1(x_1, x_2) = \mathrm{e}^{-x_1}\, \sqrt{a_1 x_2 / x_1}\, J_1(2\sqrt{a_1 x_1 x_2}),$$

where J_0 and J_1 are zero- and first-order Bessel functions, respectively. Since the dimensionless parameter a_1 is small (on the order of 10^{-3}–10^{-2}), in calculations of, e.g., the relative capillary radius, it suffices to use an asymptotic expansion of the above relation for the function r_1. As a result, we have

$$R/R_0 = 1 + a_0(l - x)\, \mathrm{e}^{-\lambda x},$$

where $\lambda = 2A/(R_0 v_0)$, $l = v_0 t$ is the depth of penetration of the solution, and $a_0 = 2Aa_1/(R_0 v_0)$ is a small parameter.

Let the size distribution of conditional (hydraulic) pores ρ with density $F(\rho)$ in the porous medium obey the lognormal law

$$F(\rho) = n_0 \exp\left[-\ln^2(\rho/\rho_0)/(2\sigma^2)\right],$$

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where ρ_0 is the distribution mode of capillary radii and n_0 is a parameter related to the dispersion and number of capillaries. Then, assuming that the dispersion and total number of capillaries in unit cross section of a rock sample are constant in the case of axisymmetric penetration of the acid solution into the formation, for changed filtration characteristics, we can write the following relations:

— for the porosity,

$$m(r) = m_0[1 + 2a_0(R_* - r)\,\mathrm{e}^{-\lambda r}]; \tag{3}$$

— for the permeability,

$$k(r) = k_0[1 + 4a_0(R_* - r)\,\mathrm{e}^{-\lambda r}]; \tag{4}$$

— for the capillary pressure,

$$p_c(s) = p_c^0\varphi(s)[1 - a_0(R_* - r)\,\mathrm{e}^{-\lambda r}]. \tag{5}$$

Here r is the current radial coordinate and R_* is the radius of the treated zone. Formulas (3)–(5) are obtained by expansion of the corresponding moments of the distribution function in the small parameter a_0 with retention of the first two terms. The indicated parameter and the constant λ are related to the parameters of the reaction-kinetics equations and the treatment method: the time of injection of the reagent T, the reagent volume Q_* and concentration C_0, and the formation thickness M. Obviously, for the radius R_* of the treated zone, the relation $R_* = \sqrt{Q_*/(\pi M m_0)}$ holds. The above-mentioned constants are as follows: $\lambda \sim (1-5)m^{-1}$ and $a_0 \sim (10^{-2}\text{–}10^{-1})m^{-1}$.

The initial formation parameters — the porosity m_0, permeability k_0, and characteristic capillary pressure p_c^0 — are constant for $r \geqslant R_*$. The dimensionless function $\varphi(s)$ included in Eq. (3) (an analog of the Leverette function) can be specified as $\varphi(s) = \sqrt{s(æ-1)/(æ-s)}$ or $\varphi(s) = \sqrt{s/(1-s)}$, where s is the oil saturation and the parameter $æ$ is usually in the range 1.1–1.3, depending on the physicochemical properties of the formation.

The system of equations commonly used in studies of immiscible fluid filtration is classified as a degenerating elliptic-parabolic system [2]. From theoretical and applied viewpoints, in deriving the basic equations describing the process of immiscible displacement, it is reasonable to take into account the compressibilities of the matrix and moving phases. In this case, as will be seen from the following, the system of equations becomes simply parabolic.

We write the mass conservation laws for oil and water:

$$\frac{\partial}{\partial t}(m\rho s) = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\rho f(s)k_f\frac{\partial p}{\partial r}\Big), \qquad \frac{\partial}{\partial t}(m\rho_{\mathrm{w}}s_{\mathrm{w}}) = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\rho_{\mathrm{w}}f_{\mathrm{w}}(s_{\mathrm{w}})k_{f\mathrm{w}}\frac{\partial p_{\mathrm{w}}}{\partial r}\Big). \tag{6}$$

Here s and s_{w} are the saturations, m is the formation porosity, ρ and ρ_{w} are the densities, p and p_{w} are the pressures, k_f and $k_{f\mathrm{w}}$ are the filtration coefficients, and f and f_{w} are the phase permeabilities; the subscript "w" refers to the water phase. According to the linear theory of elastic filtration [3], for $s = 1$, we have $m \simeq m_0(1+\epsilon_m p)$ and $\rho \simeq \rho_0(1+\epsilon_\rho p)$; similarly for $s_v = 1$, $m \simeq m_0(1+\epsilon_{mv}p_v)$ and $\rho_v \simeq \rho_{0v}(1+\epsilon_{\rho v}p_v)$, where the quantities ϵ with subscripts are experimentally determined small constants. Generally, for $s \neq 0$ and $s \neq 1$, the compression coefficients are functions of the saturation s. As a first approximation, they can treated as certain weighted average parameters. With accuracy up to the second order of smallness, we have the approximations

$$m\rho \simeq m_0\rho_0(1+\beta p), \qquad m\rho_{\mathrm{w}} \simeq m_0\rho_{\mathrm{w}}(1+\beta_{\mathrm{w}}p_{\mathrm{w}}),$$

where $\beta = \epsilon_m + \epsilon_\rho$ and $\beta_{\mathrm{w}} = \epsilon_{mw} + \epsilon_{\rho w}$. Substituting these relations into Eq. (6) and dropping the terms $\epsilon_\rho p$ and $\epsilon_{\rho w}p_{\mathrm{w}}$, which are small compared to unity, we obtain the following equations of two-phase filtration taking into account the formation piezoconductivity and the variation in the filtration characteristics in the wellbore zone:

$$m(r)\frac{\partial s}{\partial t} + \beta\frac{\partial}{\partial t}(ps) = \frac{1}{r}\frac{\partial}{\partial r}\Big(rf(s)\frac{k}{\mu}\frac{\partial p}{\partial r}\Big),$$

$$m(r)\frac{\partial s_{\mathrm{w}}}{\partial t} + \beta_{\mathrm{w}}\frac{\partial}{\partial t}(p_{\mathrm{w}}s_{\mathrm{w}}) = \frac{1}{r}\frac{\partial}{\partial r}\Big(rf(s_{\mathrm{w}})\frac{k}{\mu_{\mathrm{w}}}\frac{\partial p_{\mathrm{w}}}{\partial r}\Big), \tag{7}$$

$$p = p_{\mathrm{w}} + p_c^0\psi(r)\varphi(s).$$

Let us estimate the effectiveness of acidizing the wellbore zone for steady-state filtration taking into account the capillary locking of the water phase.

It is the case where capillary forces ($p_c^0 \neq 0$) play an important role in the filtration process and the water phase is fixed because the pressure in it is constant: $p_{\mathrm{w}} = p_c = \mathrm{const}$ and $p_c = p_0 - \Delta p$ (p_0 is the formation pressure and Δp is the specified depression). The negative role of capillary forces is illustrated by a simple example. Let

R be the well boundary on which the formation pressure $p = p_0$ and the oil saturation $s = s_0$ are conserved and the water saturation $s_\text{w} = 1 - s_0$ remains constant and equal to the initial water saturation of the field. In the case of no capillary forces and, hence, no capillary locking, the filtration flow in the formation is homogeneous, the oil inflow velocity is $q_0 = k_f f(s_0)\Delta p/\ln(R/r_b)$ (r_b is the well radius), and the water inflow velocity is $q_\text{w} \neq 0$. Conversely, if capillary forces are taken into consideration, the equality of the water and oil pressures in the well implies that $s(0) = 0$ and integrating the expression for the relative oil-inflow velocity

$$q_c = rf(s)k_f p_c^0 \frac{d(\varphi(s))}{dr}$$

over r from r_b to R and over s from 0 to s_0, we obtain

$$q_c = I(s_0)k_f p_c^0/\ln(R/r_b).$$

The water inflow is absent in this case. Thus, the degree of decrease in oil inflow is given by

$$\eta = q_c/q_0 = I(s_0)/[\varphi(s_0)f(s_0)].$$

Here and above

$$I(s_0) = \int_0^{s_0} f(s)\, d\varphi(s),$$

and if the phase permeability is specified as $f(s) = s^{3,5}$, this integral can be expressed in terms of elementary functions.

For the fields of Kazakhstan, the oil saturation s_0 is in the range 0.60–0.65. Simple calculations show that $\eta(0.6) = 0.446$ and $\eta(0.7) = 0.522$. Thus, because of capillary locking, the oil inflow to the well decreases by a factor of two. We note that the expression for the relative magnitude of the inflows η at $p_0 - p_c = p_c^0\varphi(s_0)$ does not include the range radius R and the determining factor is the initial oil saturation of the field s_0.

Naturally, for acidizing with variation in the physicochemical properties of the formation at a distance $R_* < R$ in the wellbore zone, the exponent η depends on the ratio of these quantities and the acidizing effectiveness is estimated numerically.

For an axisymmetric model of two-phase filtration based on Eqs. (6), an algorithm for calculating unsteady problems of mass transfer in the wellbore zone was developed. The model takes into account the compressibility of the formation, capillary forces, and the spatial nonuniformity of the distribution of the hydrophysical characteristics of the formation.

With allowance for the condition $s_\text{w} + s = 1$ and the capillary jump $p_\text{w} = p + p_c(s_v)$, system (6) can be converted to an equivalent system consisting, according to [2], of the parabolic equation for the water-phase pressure p_w

$$r\frac{\partial}{\partial t}m = \frac{\partial}{\partial r}\Big(r(k_\text{w}(s_\text{w}) + k(s))\frac{\partial p_\text{w}}{\partial r} + rk\frac{\partial p_c}{\partial r}\Big) \qquad (r_b < r < L) \tag{8}$$

and the hyperbolic transfer equation for the water saturation (s_w)

$$r\frac{\partial}{\partial t}(ms_\text{w}) = \frac{\partial}{\partial r}\Big(rk_\text{w}\frac{\partial p_\text{w}}{\partial r}\Big), \qquad m = m_0(r) + \delta p_\text{w}, \quad r_b < r < L. \tag{9}$$

The following boundary and initial conditions are specified:

$$p_\text{w}\Big|_{r=r_b} = p_\text{w}^0, \qquad p_\text{w}\Big|_{r=L} = p_\text{w}^1, \qquad s_\text{w}\Big|_{r=r_b} = s_\text{w}^0, \qquad s_\text{w}\Big|_{r=L} = s_\text{w}^1, \qquad p_v\Big|_{t=0} = p_0, \qquad s_v\Big|_{t=0} = s_0. \tag{10}$$

This problem was solved using an iterative process, which can be written in differential form

$$\frac{\partial}{\partial t}(rm^n) = \frac{\partial}{\partial r}\Big(r(k_\text{w}(s_\text{w}^{n-1}) + k(1 - s_\text{w}^{n-1}))\frac{\partial p_\text{w}^n}{\partial r}\Big) + F^{n-1},$$

$$\frac{\partial}{\partial t}(rm^n s_\text{w}^n) = \frac{\partial}{\partial r}\Big(rk_\text{w}(s_\text{w}^n)\frac{\partial p_\text{w}^n}{\partial r}\Big), \qquad m^n = m_0(r) + \delta p_\text{w}^n.$$

Here n is the iterative step number and $F^{n-1} = \dfrac{\partial}{\partial r}\Big(rk(1 - s_\text{w}^{n-1})\dfrac{\partial p_c(s_\text{w}^{n-1})}{\partial r}\Big)$.

In the nth iterative step, the pressure in the water phase is found from the first equation and the water saturation is determined from the second equation. If the conditions $\max\limits_{r_b<r<L}|s_\text{w}^n - s_\text{w}^{n-1}| < \varepsilon_1$ and $\max\limits_{r_b<r<L}|p_\text{w}^n - p_\text{w}^{n-1}| < \varepsilon_2$ are satisfied, the iterative process is terminated.

In numerical finite-difference calculations, we used conservative implicit difference schemes, which were solved by a sweep method [4]. In each time layer, a difference solution was found using the iterative algorithm described above. With a choice of rather small steps in time, this algorithm converges and allows the problem to be solved in the general formulation.

In the case of a rigid filtration regime $\delta = 0$, Eq. (8) implies that the total velocity

$$V = -\Big(rk_{\rm w}(s_{\rm w})\,\frac{\partial p_{\rm w}}{\partial r} + rk(1-s_{\rm w})\,\frac{\partial p}{\partial r}\Big)$$

is a function of only time $V = V(t)$, and in this case, the computational algorithm can be simplified [2]. The approximate value of the total velocity V^n in the nth iterative step is determined from the formula

$$V^n = -\int\limits_{r_b}^{L} r^{-1}(k_{\rm w}(s_{\rm w}^{n-1}) + k(1-s_{\rm w}^{n-1}))^{-1}\,dr\,[p_{\rm w}^1 - p_{\rm w}^0 + \Phi(s_{\rm w}^1) - \Phi(s_{\rm w}^0)].$$

Here the function Φ is defined by the equality

$$\Phi = \int\limits_0^{s_{\rm w}} \frac{k(1-s)}{k_{\rm w}(s) + k(1-s)}\,\frac{\partial p_c(s)}{\partial s}\,ds.$$

If the dependence of the equation coefficients on the saturation has the form

$$k_{\rm w}(s_{\rm w}) = k_0 s_{\rm w}^2, \quad k(1-s_{\rm w}) = k_0(1-s_{\rm w})^2, \quad p_c(s_{\rm w}) = p_c^0\sqrt{(1-s_{\rm w})/s_{\rm w}} \tag{11}$$

the value of Φ can be computed explicitly.

Using the representation

$$rk_{\rm w}(s_{\rm w})\,\frac{\partial p_{\rm w}}{\partial r} = -\frac{k_{\rm w}(s_{\rm w})}{k_{\rm w}(s_{\rm w}) + k(1-s_{\rm w})}\,V - \frac{rk_{\rm w}(s_{\rm w})k(1-s_{\rm w})}{k_{\rm w}(s_{\rm w}) + k(1-s_{\rm w})}\,\frac{\partial p_c}{\partial r},$$

we numerically solve the nonlinear parabolic equation

$$\frac{\partial}{\partial t}\,(rm^n s_{\rm w}^n) = -\frac{\partial}{\partial r}\Big(\frac{k_{\rm w}(s_{\rm w}^n)}{k_{\rm w}(s_{\rm w}^n) + k(1-s_{\rm w}^n)}\,V^n + \frac{rk_{\rm w}(s_{\rm w}^n)k(1-s_{\rm w}^n)}{k_{\rm w}(s_{\rm w}^n) + k(1-s_{\rm w}^n)}\,\frac{\partial p_c(s_{\rm w}^n)}{\partial r}\Big),$$

and determine the next iterative approximation for the water saturation.

The model was implemented numerically in C++. The program provides for information input in a conversational mode and a graphical representation of calculations. Using this program, one can solve the problem in the axisymmetric and one-dimensional formulations with first- and second-kind conditions on the right boundary ($r = L$). The hydrophysical characteristics of the formation can vary depending on the distance to the well and are specified as piecewise constant functions. The local variation in the formation properties near the well due to acidizing is specified according to formulas (3)–(5). The calculation results are displayed as tables or plots. After data analysis and updating, the calculations can be continued using the obtained results as new initial data.

As is known [3], in immiscible filtration problems incorporating capillary forces, it is theoretically assumed that the normal derivative of the saturation increases without bound as the coordinate approaches the radius of the operating well. Therefore, in the numerical implementation of the problem, we used a nonuniform mesh with refinement in the neighborhood of the well. The conservatism of the scheme allows a decrease in the effect of the saturation approximation error in the wellbore zone.

The developed program was used to calculate the oil inflow to the

exhaust well with variation in the type of model and the acidizing of the wellbore zone. The equation coefficients were specified as (11). The calculations were performed for the following data of the axisymmetric problem.

The constant parameters of the problem were as follows: well radius $r_b = 0.1$ m, porosity $m = 0.15$, filtration coefficient $k_0 = 0.2$ m/days, length of the region $L = 30$ m, initial water saturation $s_{\rm w} = 0.4$, ratio of the water and oil viscosities $\mu_0 = 0.2$, decrease in the well head pressure compared to the formation head pressure $\Delta p = p_{\rm w}^1 - p_{\rm w}^0 = 20$ m ($p_{\rm w}^0 = 0$). In the calculations, the characteristic values of the capillary jump (p_c^0) and the formation compressibility factor (δ) were varied.

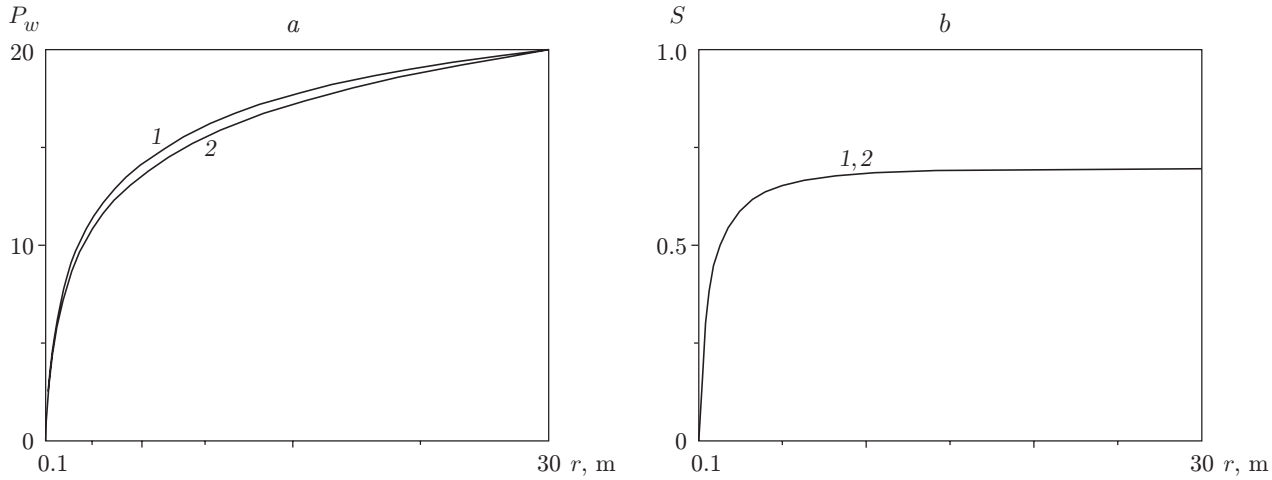

Fig. 1. Pressure (a) and oil saturation (b) distributions ($\delta = 0.0001$ and $p_c^0 = 0.8$) for $t = 10$ (1) and 100 days (2).

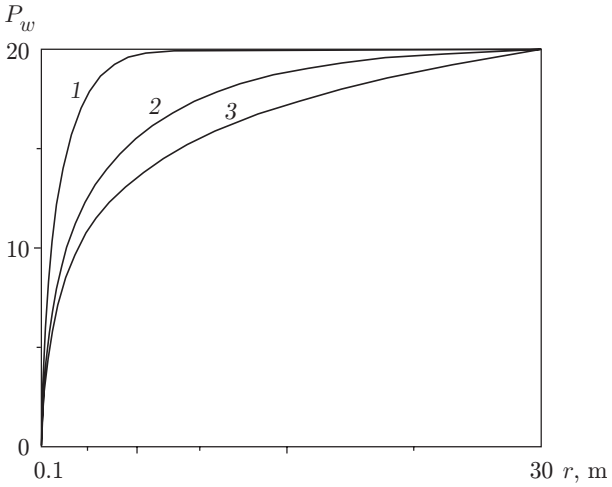

Fig. 2. Pressure distribution ($\delta = 0.005$) for $t = 0.5$ (1), 10 (2), and $t = 100$ days (3).

TABLE 1

t, day	$q/q_{\rm w}$ ($\delta = 0.0001$, $p_c^0 = 0$)	$q/q_{\rm w}$ ($\delta = 0.0001$, $p_c^0 = 0.8$)
0.5	0.335/0.734	0.159/0.798
10	0.321/0.708	0.147/0.778
100	0.321/0.708	0.147/0.781

TABLE 2

t, day	q/q_{w} ($p_c^0 = 0$)	q/q_{w} ($p_c^0 = 0.8$)	q^0/q_{w}^0 ($p_c^0 = 0$)	q^0/q_{w}^0 ($p_c^0 = 0.8$)
0.5	0.417/0.914	0.234/0.974	1.245/1.245	1.472/1.221
10	0.399/0.881	0.218/0.946	1.243/1.244	1.483/1.216
100	0.398/0.883	0.218/0.950	1.243/1.244	1.483/1.216

TABLE 3

t, day	q/q_{w} ($\delta = 0.005$, $p_c^0 = 0$)	q/q_{w} ($\delta = 0.005$, $p_c^0 = 0.8$)
0.5	0.566/0.993	0.317/1.096
10	0.389/0.730	0.186/0.808
100	0.389/0.730	0.186/0.808

Calculated production rates of the untreated well are given in Table 1, where the numerator corresponds to the oil flow rate and the denominator to the water flow rate.

In the numerical modeling of the two-phase flow with well acidizing, the physical parameters of the formation were $\delta = 0.0001$, $p_c^0 = 0$, and $p_c^0 = 0.8$ and the parameters of the treated wellbore zone were $a_0 = 0.05$, $\lambda = 1$, and $R_* = 2$ (Table 2). The production rate of the treated well increases irrespective of whether capillary forces are taken into account ($p_c^0 = 0.8$) or not ($p_c^0 = 0$). The last two columns of Table 2 give the flow rate ratio for the cases with and without acidizing (q^0 refers to oil and q_{w}^0 refers to water).

Figure 1 gives pressure and oil-saturation distributions for a weakly compressible formation without acidizing. For times larger than 10 days, the oil saturations practically do not change.

We should particularly note the effect of the formation compressibility. Unlike in the rigid filtration regime, the solutions of elastic problems depend substantially on the formation pressure dynamics. This can be clearly seen in the case of no capillary jump. For a homogeneous water saturation of the formation at the initial time $s\Big|_{t=0} = s_0 = \mathrm{const}$) and the boundary condition $s\Big|_{r=L} = s_0$, the system of Buckley–Leverette equations ($\delta = 0$) has a solution in the form of the constant $s_{\mathrm{w}}(r,t) = s_0$. However, for the elastic filtration regime, the solution has an absolutely different form: the water saturation and oil saturation are nonmonotonic time-dependent functions. The compressibility effect is also manifested in the presence of a capillary jump; it is especially strong for large values of the compressibility factor ($\delta = 0.005$), which can be due to the presence of a gas phase in the formation. In this case, passage of the pressure release wave near the well sharply changes the oil saturation distributions, and in addition, the pressure gradient near the well becomes high enough (Fig. 2). These features of the process may be responsible for the higher values of oil inflow to the well at the initial time (Table 3) compared to the results obtained for weakly compressible formations ($\delta = 0.0001$) (see Table 1).

The calculation results lead to the following general conclusion. Acidizing of the wellbore zone of a producing well can be an effective method for increasing the oil inflow. Naturally, the degree of effectiveness of this method depends on the treatment technology used and the physicochemical parameters of the formation. For example, from the results presented in Table 2, it is obvious that a decrease in the negative effect of capillary forces plays an important role in increasing the oil inflow (by a factor of about 1.5).

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